

# NBS TECHNICAL NOTE 685

U.S. DEPARTMENT OF COMMERCE/National Bureau of Standards

## A SYSTEM FOR CALIBRATING LASER POWER METERS FOR THE RANGE 5-1000 W

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A SYSTEM FOR CALIBRATING LASER  
POWER METERS FOR THE RANGE 5-1000 W

E. D. West\* and L. B. Schmidt

This technical note describes the National Bureau of Standards measurement system for calibration and test of laser power and energy devices at power levels up to 1 kW and total energies of several kJ. The main parts of the system are two electrically calibrated calorimeters and several beam splitters of different materials. The estimated limits of systematic error in measurements with the two calorimeters are  $\pm 1.9\%$  for one and  $\pm 2.0\%$  for the other. The two calorimeters have the same precision, and the standard deviation of an electrical calibration is the same as the standard deviation of a laser energy measurement, indicating that the laser beam does not affect the precision of the measurement. The standard deviation of an energy measurement depends on the total energy to the calorimeter and is about 0.8% at 1 kJ.

Key words: Calorimeter; laser calorimeter; laser energy; laser power

1. GENERAL REQUIREMENT FOR A CALIBRATION SYSTEM

The main elements of our laser power, or energy, calibration system are a laser source, a shutter for interrupting the beam, a beam splitter, a monitor calorimeter, and a calorimeter whose response coefficient has been evaluated. The devices to be calibrated are compared to one of these calorimeters by substituting the devices to be calibrated, one at a time, for one of the calorimeters in the calibration system. A typical experimental arrangement for calibrations is shown in the diagram in figure 1.

The basic requirement of the elements of a calibration system is stability of their intrinsic characteristics. In a laser power meter calibration the output power of the laser source must be reasonably constant during the injection time to eliminate serious error due to averaging. The response of both of the calorimeters to specified energy inputs must repeat, at the predicted accuracy, to maintain the accuracy of the calibration system. Equipping the calorimeters so that known amounts of electrical energy can be substituted for laser energy provides a non-laser check on the stability of their response to the electrical energy. The stability of the beam splitter ratio -- the ratio of the energies in the transmitted and the reflected beams -- is the first consideration in selecting a beam splitter. Long term stability of the ratio of beam splitter is assured when the reflecting surfaces are of high quality, and the ratio is determined by the index of the refraction of the material.

\*Presently with Calorimetrics, Inc., Boulder, Colorado.

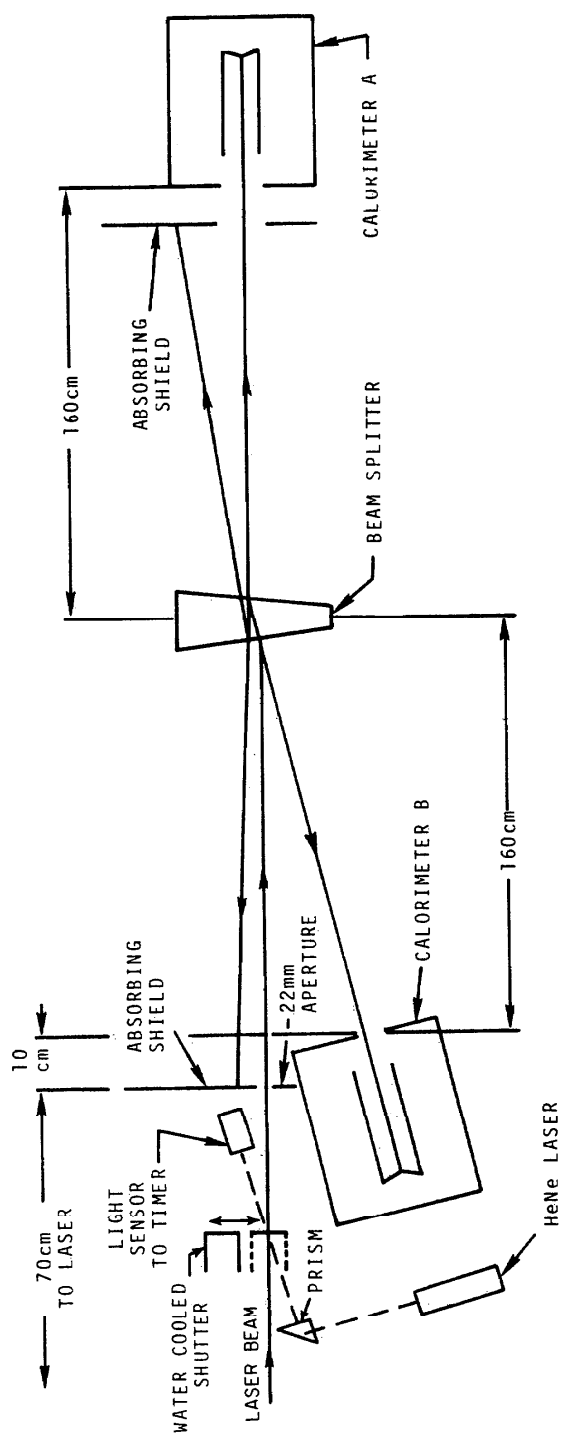


Figure 1. A typical beam splitter arrangement. The laser beam is admitted to the system by removing the water-cooled shutter. The beam passes through a 22 mm aperture to a wedge beam splitter. The transmitted beam goes into calorimeter A and the first surface reflection to calorimeter B. Other reflections are terminated on absorbing shields. A helium-neon beam is used to actuate a timer when the shutter is opened and closed.

After the calorimeters of a system are electrically calibrated and their absorptances are determined, the absolute beam splitter ratio is determined as an energy ratio, which is a property of the beam splitter [1]\*. At a particular wavelength the beam splitter ratio should be constant for all combinations of calorimeters placed in the transmitted and reflected beams. Interchanging calorimeters in this manner provides a check on the stability of the beam splitter, and a cross-check on systematic and random errors of the calibrating system.

The selection of the beam splitter is also influenced by the energy ranges of the calorimeters and the power or energy output of the laser. The energies provided in the transmitted and reflected beams must be values that can be measured by the calorimeters to their estimated accuracy. The power, or energy in the transmitted beam should be as large as possible if one desires to perform calibrations near the maximum output level of the laser.

The injection time of the laser beam must be measured when power meters are to be calibrated by comparison to a calorimeter. The method of injecting the laser beam into the system must be designed to include the measurement of this injection time and this measurement must be traceable to a time standard.

## 2. THE CALIBRATING SYSTEM

NBS has developed a laser power and energy measurements system with a 5-to-1000-watt range for the calibration of devices for this power range. The basic measurement problem is the determination of calibration factors and their associated uncertainties in terms of power or energy as defined by the international system of units.

The main elements of the NBS calibrating system conform to the requirements discussed previously. One form of an experimental arrangement for calibrations is shown in the diagram in figure 1. The experimental arrangement is the same principle as the system for lower power [2].

The laser used in the experimental arrangement of figure 1 required that the laser beam injection time be controlled and measured external to the laser. This was done by using a water cooled shutter to admit the laser beam into the system for a predetermined time interval. A red helium neon laser beam is applied to a light sensor whose output voltage triggers a time interval counter used to measure the time. The time interval counter is equipped with a crystal-controlled oscillator that is checked against a 100 kHz standard frequency supplied by the Time and Frequency Division of

\*Figures in brackets indicate the literature references at the end of this paper.

NBS. Another laser used with the calibration system is equipped to allow the laser beam to be turned on for a specified time interval by an electronic gate. The time interval of the gate is measured by the above time interval counter.

This system has been evaluated only at 10.6  $\mu\text{m}$ . The beam splitters were selected primarily for the stability of the ratio of the energies in the transmitted and reflected beams. The power range of this system adds further requirements in the selection of beam splitters. Beam splitters for calibration work with 10.6  $\mu\text{m}$  laser beams have been studied by Franzen [1]. For general use he recommends zinc selenide beam splitters, which transmit 70% of the incident beam. When greater transmission is desired, he suggests potassium chloride beam splitters which transmit 92% of the incident power. To extend the system capability to a new wavelength, it is necessary to acquire the laser and measure the absorptance of the calorimeters at the laser wavelength.

The electrically calibrated calorimeter is the element in the measurement system that provides the link to the international system of units. The standard volt and ohm have been determined in terms of the base units to accuracies of parts per million and time measurements are accurate to parts in  $10^{13}$ . These standards are therefore much more accurate than calorimetry requires, and the methods of using them can be relaxed considerably in the interest of efficiency and convenience.

The electrically calibrated calorimeter is not itself a standard. Its relationship to energy resembles that of a potentiometer to voltage or of a bridge to resistance. It is the means of comparing electrical energy to laser energy.

The theory of the calorimetric measurement is summarized by two equations [2]. The first relates the electrical or laser work  $W$  done on the calorimeter to the energy equivalent or calibration factor  $E$  multiplying a corrected temperature rise

$$W = E[T_F - T_I + \epsilon \int \{T - T(\infty)\} dt] , \quad (1)$$

where  $T$  is the temperature observed at time  $t$ ,  $T_F$  and  $T_I$  are temperatures characterizing respectively the final and initial rating periods [3,4], the limits of the integral are taken between the times corresponding to these temperatures,  $T(\infty)$  is the steady-state or convergence temperature, and  $\epsilon$  is the cooling constant. The last two quantities are obtained from the equation

$$dT/dt = - \epsilon [T - T(\infty)] . \quad (2)$$



This equation is integrated to give

$$T - T(\infty) = [T_I - T(\infty)] \exp [-\epsilon(t - t_I)] \quad (3)$$

for the initial rating period and a similar equation for the final rating period with  $T_I$  and  $t_I$  replaced by  $T_F$  and  $t_F$ . The rating period data are fit by least squares to obtain the quantities  $T_F$ ,  $T_I$ ,  $\epsilon$  and  $T(\infty)$  required in eq. (1). More detailed descriptions of the measurement theory and calculations are given in [3,4].

### 3. THE K1 CALORIMETERS

The two electrically calibrated calorimeters in the NBS system are of the same design, with laboratory designations K1-1 and K1-2. The construction of the calorimeters is illustrated by the cross-sectional diagram in figure 2. The apparatus is 36 cm long over-all and 15 cm in diameter. The calorimeter proper consists of a copper absorbing cavity 25 cm long by 4 cm in diameter attached to and enclosed by a copper shield to enhance the equivalence of the electrical and laser sources. The main thermal contact between absorbing cavity and shield is through a copper ring soldered about half way between the ends of the calorimeter proper. The significance of the location of the copper ring is discussed in section 6. The calorimeter is calibrated by means of an electrical heater wound on the outside of the absorber and the temperature is measured with a resistance bridge, as described by Maier [5], wound on the shield. The calorimeter proper is supported on three stainless steel tubes. These tubes are soldered to threaded brass caps which are threaded into the temperature controlled jacket that is 1.3 cm thick. These supports are equally spaced on the perimeter of the temperature controlled jacket. Spaces between the calorimeter proper and the jacket and between the absorber and the shield are about 1 cm, a space small enough to avoid convection and the associated non-linearities but large enough to hold the thermal conductance of the air to a reasonable value. The temperature controlled jacket is supported on three stainless steel tubes mounted and spaced equally on the perimeter of the supporting ring. The construction of these support tubes is similar to those mentioned above. The temperature controlled jacket has two parallel spiral grooves cut along its length. Another resistance-thermometer bridge and a heater are wound in the grooves which allows temperature control of the jacket. A tube leads into the calorimeter proper for cooling it by an air current to shorten the cool-down time between experiments. The calorimeter proper and the jacket are enclosed in an aluminum cover anodized black to absorb stray laser radiation.

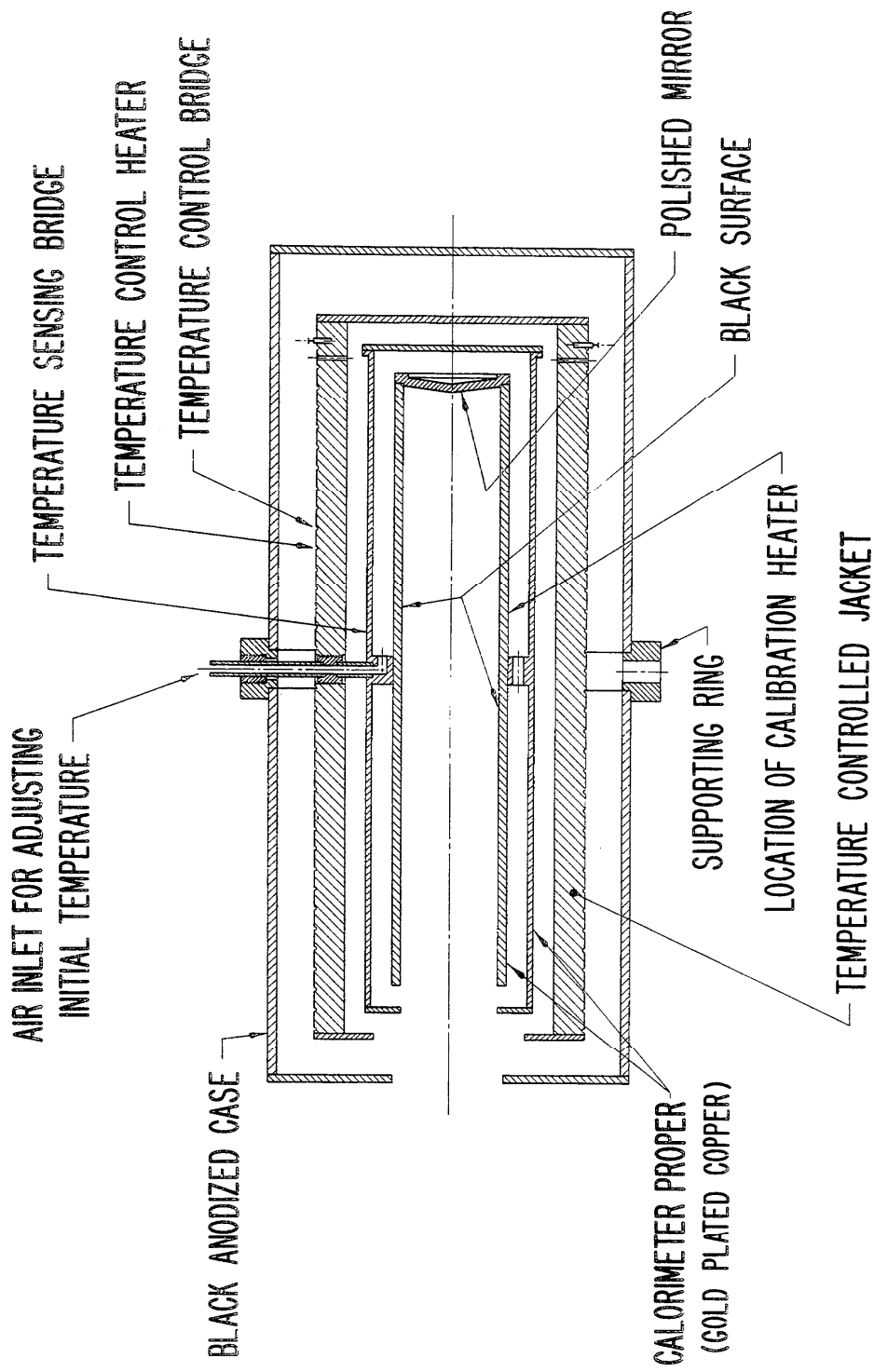


Figure 2. Cross-sectional diagram of the K1 calorimeters. A laser beam entering along the axis strikes a conical mirror and is spread out along the inner blackened walls of the calorimeter proper. The consequent rise in temperature is measured by a resistance bridge. The calorimeter proper is mounted in a constant-temperature jacket to satisfy theoretical assumptions needed to evaluate heat lost from the calorimeter proper.

The right end of the absorber is closed by a polished gold-plated spherical mirror, which reflects the incident laser power to the black-painted shield. The mirror has an apex angle of  $166^\circ$  and is replaceable. It serves to spread the incident power so that the power density will be below the damage threshold of the 3M Nextel\* black paint, about  $200 \text{ W/cm}^2$ . Care in using the laser is necessary to keep the power density below the damage threshold of these surfaces.

The calibrating electrical heater consists of 100 ohms of 0.25 mm diameter manganin wire in a bifilar winding covering about 6 cm of the absorber between the copper ring and the mirror mount. The heater is operated at power levels between 5 W and 1 kW for a total energy input of 300 J to 1 kJ (but 1.5 kJ at 1 kW). The copper heater leads are 0.4 mm in diameter with a length of about 3.8 cm between the shield and the jacket. For thermal contact the leads are attached to the shield and the surroundings with Epon 815 manufactured by Shell Chemical Co. The connection to the calorimeter heater is made inside the shield at the surface of the absorber. Potential leads are attached midway between the shield and the jacket.

The resistance bridge used for temperature measurement is wound on the shield near the copper ring. The bridge consists of two copper arms and two manganin arms each of 100 ohms resistance at the balance temperature, which is chosen to be just below the steady-state temperature  $T(\infty)$  so that the temperature reading is always positive. The bridge is supplied by a 20 mA current source constant to 0.005%. If the four arms are equal, the bridge output  $E_o$  is given by  $E_o = 1/2 IR\beta\Delta T$ , where  $I$  is the constant current,  $R$  is the resistance per arm,  $\beta$  is the temperature coefficient of the copper, and  $\Delta T$  is the difference between the actual temperature and the temperature at balance. The formula implies a bridge output linear with the temperature to the extent that  $\beta$  is constant. The real question of the linearity of the calorimeter based on the actual thermometer is discussed with the electrical calibration data.

The temperature of the temperature-controlled jacket must be brought to the control point before the calorimeter can be operated. The theory requires this temperature to be invariant with time during an experiment. If this temperature varies appreciably, the heat exchange will be incorrectly evaluated.

The controlled temperature is sensed by a resistance bridge that is the same type as the one used for the calorimeter-proper temperature measurement. The bridge is fastened in a groove with Epon 815 epoxy resin. It balances at about  $43^\circ \text{C}$  and is powered with 20 mA dc. The output is amplified by an operational amplifier having a low offset voltage and gain stabilized with

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\*Certain materials are identified in this paper in order to specify adequately the experimental procedure. In no case does such identification imply recommendation or endorsement by the National Bureau of Standards, nor does it imply that the material or equipment identified is necessarily the best available for the purpose.

a feedback resistor. The amplified signal is fed to intermediate operational amplifiers which provide proportional and integral (reset) control to an output transistor. This transistor regulates the current in a 30- $\Omega$  control heater wound in a groove near the control thermometer.

The temperature is controlled to  $\pm 0.1$  mK as calculated from the gain of the operational amplifier and the bridge parameters. Experience indicates that temperature control is not the limiting factor in the precision of the measurement.

The temperature-controlled jacket does not form a complete thermal enclosure for the calorimeter proper. Some heat is transferred to the room through the opening for the laser beam. To this extent the room is a part of the thermodynamic surroundings. In principle, the variation in room temperature would set a lower limit on the laser energy to be measured accurately in isoperibol calorimeters, but we have not yet established that temperature variation does indeed set the limit.

The output of the thermometer that measures the temperature of the calorimeter proper is amplified by a dc amplifier linear to 0.01%. This amplifier is epoxied to the rear cover of the temperature controlled jacket. The output of the amplifier is read by a digital voltmeter. At 10 sec intervals, the voltmeter reading is transferred to a data coupler and then punched on a paper tape.

#### 4. USE OF THE K1 CALORIMETERS

In preparing for an experiment, the dc amplifier is allowed to warm up while the temperature control is established. About two hours after the temperature of the temperature controlled jacket is under control the rate of change of the temperature of the calorimeter proper in eq. (2) becomes small enough to make a measurement. We keep the rate of change in the initial rating period less than about ten percent of the rate of change in the final rating period, because we believe that the increase in internal energy can be evaluated more accurately than the heat exchange. If the calorimeter proper is cooling rapidly in the initial rating period, the heat exchange term for the experiment will be large.

The experiment cannot be started until the rate of change of temperature of the calorimeter proper is small as defined above, but one hour after the preceding experiment will usually prove more than adequate for this purpose. In any case, this point is checked in the computer data analysis.

When one is satisfied that the temperature difference is small enough so that eq. (2) holds, a number of data points of the thermometer response are logged (20 to 30) and then either the electrical, or the laser input is made. We frequently log a minimum of 31 points at 10 sec intervals.

After the input, enough points are taken to make sure that 20 to 30 points are in the final rating period after higher order exponentials have become negligible and eq. (2) again can be applied. The final rating period can usually be started about 400 sec after the input is stopped. The computer program [3] prints deviations of individual points from the integral form of the eq. (2) to show that the higher order exponentials have become negligible before the final rating period is started. The initial testing of the calorimeters includes the determination of the data point at which the final rating period can be started.

## 5. ELECTRICAL CALIBRATIONS

The electrical calibrating circuit diagram in figure 3 is based on principles long in use in calorimetry. The diagram is included to facilitate discussion of the errors in the measurement. Electrical calibrations are carried out using the calorimeter-proper heater as a four-terminal resistor, measuring the dc current in the heater, the voltage across it, and the time the power is on. The heater current is determined from the voltage across a standard resistor. Different standard resistors are used to accommodate different heater currents. The voltages required for the power computation are measured with digital voltmeters.

The power source for calibration is a dc supply operating up to 317 volts and 3.15 amps. The current is regulated to 0.1%. This regulation is better than necessary and is a matter of convenience.

The time is read by a counter-timer with a built-in crystal-controlled oscillator. The oscillator is checked against a 100 kHz standard frequency supplied by the Time and Frequency Division of NBS. The timer is connected in parallel with the calibrating heater and is triggered by the voltage across the heater. Any leakage current in the timer is by-passed around the standard resistor and calorimeter heater to avoid a systematic error in the heater power.

Electrical calibrations have been carried out for two calorimeters K1-1 and K1-2 of the type shown in figure 2. The calibration factor  $E$  is obtained by a least squares fit of the corrected temperature rise  $\Delta T_i$  and the energy  $W_i$  according to the equation  $\Delta T_i = W_i/E$ . For K1-1, the electrical calibration is 2.262 J/mV, based on 36 electrical calibrations taken over a period of two years. The standard deviation of the mean of this factor is  $1.5 \times 10^{-3}$  or 0.066%. For K1-2, the calibration factor is 2.276 with the standard deviation of the mean of 0.058% based on 35 measurements over the same period. The difference in the standard deviation is not significant ( $F = 1.3 < 1.9 = F_{.95} \{35,34\}$ . See Natrella [6], Chapter 4.)

The residuals from the straight-line fit must be examined for systematic deviations from a straight line, indicating non-linearity. The maximum

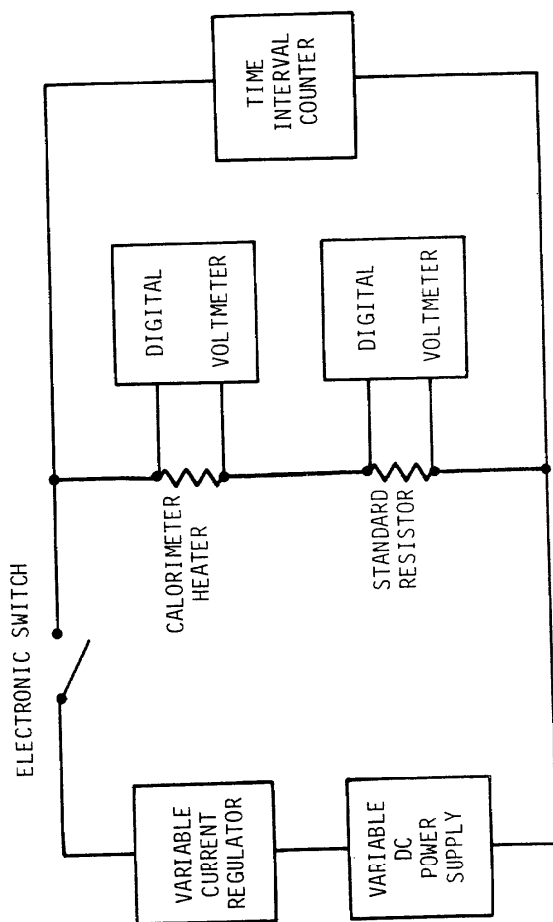


Figure 3. Diagram of the electrical calibrating circuit. Current from a dc supply is switched to the calorimeter-proper heater and a series standard resistor. The voltages across the two are measured to get the electrical power. The counter registers the time interval over which the switch is closed.

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temperature rise for Kl-1 and Kl-2 is about 3°K and in this range we find no systematic deviation from linearity.

The straight-line fit takes the energy input to be without significant random error and calculates a standard deviation  $s_{\Delta T}$  for the corrected temperature rise. For Kl-1,  $s_{\Delta T} = 3.65$  mV and for Kl-2  $s_{\Delta T} = 3.58$  mV. From this standard deviation by propagation of error formulas we obtain the standard deviation of the calibration factor and the standard deviation  $s_E$  of individual determinations of the calibration factor. The standard deviation  $s_E$  depends on the energy input  $W$  according to the formula

$$s_E = (E^2/W) s_{\Delta T} . \quad (4)$$

This formula is derived using Ku's propagation-of-error formulas [7]. The quantity  $s_E$  may be looked upon as the standard deviation we would obtain if we made a large number of measurements at the same energy input  $W$ . The standard deviation  $s_E$  increases as the energy  $W$  becomes smaller. This behavior is consistent with the physics of the measurement from which we would expect very low energy experiments to get lost in the random error.

The standard deviation  $s_E$  is important because it is an index of the random error of an electrical energy measurement and therefore bears on a laser energy measurement with the calorimeter. It is taken into consideration later in estimating the error in such a measurement.

Systematic errors in the electrical measurements come from the instrumentation of figure 3. Three different standard resistors have been used, a nominal one ohm and two nominal one-tenth ohm. The estimated limits of systematic error are  $\pm 0.005\%$  for the one ohm resistor and one of one-tenth ohm resistors and  $\pm 0.01\%$  for the other one-tenth ohm resistor. The voltages are read by two digital voltmeters calibrated against a standard cell with a guarded laboratory potentiometer. The estimated limits of systematic error in the calibration of the voltmeters are  $\pm 0.02\%$ . The standard cell and the standard resistors are calibrated by the Electricity Division of NBS. The allocation of power generated in the heater leads constitutes the largest systematic error in the electrical calibration. The length of each lead is 1.9 cm between the outer surface of the calorimeter proper and the temperature-controlled jacket with voltage taps at the midpoints, but the leads are brought through a radial hole in the 1.6 mm wall of the shield of the calorimeter proper. The exact size of this hole is not known and the 1.6 mm distance may be too short to bypass to the shield the heat generated in the lead between the calibrating heater and the shield. The length of each lead from the heater to the voltage tap at the midpoint above is 6.9 cm. If this heat is equally divided between the calorimeter proper

and the temperature-controlled jacket, the heater voltage taps are positioned wrong by  $(6.9/2) - (1.9/2) = 2.5$  cm. The heater leads are 0.4 mm diameter copper and the error corresponding to  $2 \times 2.5$  cm is 0.34 ohm or 0.34% of an energy input. The error is in the direction of making the observed number of joules greater than actual. If we were confident that no heat is bypassed to the outer calorimeter cylinder, we would correct the calibration factor in joules per millivolt by multiplying by .9983 and estimating limits of systematic error of  $\pm 0.17\%$ . If the hole in the shield is small enough, the heat will be bypassed to the shield and the estimated systematic error depends only on the inaccuracy of locating the voltage taps at the mid-points and the conduction of the voltage taps themselves.

The time interval counters used to measure the time of the electrical inputs are compared to a time standard accurate to one part in  $10^{10}$  with an error of one least count or one part per million of a full-scale reading. The timing error is therefore negligible.

The estimated limits of systematic error in an electrical calibration are summarized in table I.

Table I. Estimated limits of systematic errors in electrical quantities.

<u>Source of Error</u>	<u>Estimated Limits</u>
Standard cell	0.005%
Calibration of digital voltmeters	.04
Standard resistor	.01
Heater leads	.34
Time	<u>.000</u>
Sum (rounded)	.40%

Simply summing the systematic errors is a very conservative way of combining them. (Propagation-of-error formulas allow summing percentages in this case.)

The resistance of the electrical heater provides a convenient means of monitoring the electrical circuit, figure 3, including the precision of the two voltmeters and the heater itself. A significant change in the heater resistance serves as an alert to trouble, possibly in grounding of the heater or its leads. For calorimeter K1-1, the heater resistance is 101.12 ohms with a standard deviation of the mean of 0.07 (32 measurements). For K1-2, the resistance is 101.42 and the standard deviation of the mean of 0.08.



## 6. ERRORS DUE TO NON-EQUIVALENCE OF SOURCES

The energy supplied to a calorimeter is evaluated by means of the First Law of Thermodynamics. This law equates the electrical or electromagnetic work done on the calorimeter proper to the change in the internal energy plus the heat transferred to the surroundings. The change in internal energy is proportional to  $T_F - T_I$  as in eq. (1) and is properly evaluated by the techniques of section 1 independent of the location of the source. The heat exchange term  $\epsilon \int (T - T_\infty) dt$  in eq. (1) fails to take into account the heat exchange due to higher order exponentials [4]. The error is proportional to the input  $W$  and the constant of proportionality depends on the geometric location of the source in the calorimeter proper. Laser and electrical sources are therefore not exactly equivalent in the errors they cause in accounting for heat exchanged with the surroundings. To the extent that heat from both sources can be made to flow to the surface of the calorimeter proper by way of the same isothermal region, this region acts as a secondary source and makes the constant of proportionality the same for both sources. In figure 2, the copper ring is designed to serve as the secondary source.

To check the design and determine if the copper ring is effective, experiments were carried out with two different heaters. A small heater was wound on the mirror and electrical calibrations made with it were compared to the regular electrical calibrations. The mirror location is thermally quite different from that of the calibration heater, which is located along the wall where the laser beam is absorbed. Heat from the mirror heater travels farther to reach the copper ring and allows greater opportunity for heat to bypass the ring by direct flow to the outer shield of the calorimeter proper. Comparison of the calibrations using the two different heaters should therefore give a worse-case estimate of the systematic error from this source.

The wire size of the mirror heater limited the maximum power level of the electrical calibrations to 6 W, which is at the low end of the range for the calorimeter. As a consequence, the random errors of the data from the mirror heater calibrations were somewhat greater than the random errors of the data from the regular calibrations.

For calorimeter K1-1, the comparison is based on twenty-five regular calibration experiments, which give a mean calibration factor of 2.2609 J/mV, and a standard deviation of the mean of 0.002, and eight calibrations with the mirror heater, which give a mean of 2.2610 and a standard deviation of the mean of 0.007. The difference between the two calibration factors is obviously not statistically significant. (The 99% confidence interval is  $\pm 0.0241$ .)

For calorimeter K1-2, twenty-one regular calibrations give a mean of 2.2793 J/mV and a standard deviation of 0.0017; eight calibrations with the mirror heater give 2.2736 and a standard deviation of 0.0059. The difference between the averages is again not statistically significant (99% confidence limits  $\pm 0.0204$ ).

The mean calibration factors used in this comparison differ slightly from the current values given earlier. Although the conclusions reached would be the same, we feel that the data presented better represent the state of the measurement system at the time the mirror heater calibrations were made.

From the preceding calculations we estimate the limits of systematic error due to the non-equivalence of sources to be  $0.0001 \pm 0.0241$  for K1-1 and  $0.0057 \pm 0.0204$  for K1-2, where the stated errors are the 99% confidence intervals.

## 7. ABSORPTANCE OF THE CAVITY

The absorptance was measured by the method of West and Schmidt [8]. In essence, the method consists of placing an auxiliary calorimeter over the opening in the absorbing cavity. The auxiliary calorimeter is a shallow cup with a small hole in the center. A laser beam enters the absorbing cavity through this hole. Laser power not absorbed by the cavity is measured by the auxiliary calorimeter.

The reflectance  $1 - \alpha$  for the cavity is given by their equation

$$1 - \alpha = \frac{P_2 / \alpha_2 F_2}{P_1 + P_2 / \alpha_2 F_2} , \quad (5)$$

where  $\alpha$  is the absorptance factor of the cavity,  $P_1$  is the power to the calorimeter proper,  $P_2$  is the power to the auxiliary calorimeter,  $\alpha_2$  is the absorptance factor of the auxiliary calorimeter and  $F_2$  is the fraction of radiation from the cavity that strikes the auxiliary calorimeter.  $F_2$  allows for the hole for the laser beam. The powers  $P_1$  and  $P_2$  are obtained by simultaneous solution of eqs. (6) below relating the temperature  $T_i$  of the two calorimeters to the powers through a set of four experimentally-determined transfer functions  $I_{ij}$

$$T_1(t) - T_1(\infty) = I_{11}(t)P_1 + I_{12}(t)P_2 \quad (6)$$

$$T_2(t) - T_2(\infty) = I_{21}(t)P_1 + I_{22}(t)P_2 ,$$

where  $T_1(\infty)$  and  $T_2(\infty)$  are the temperatures in the steady state with  $P_1 = P_2 = 0$ . The  $I_{ij}$  are obtained by alternate electrical inputs with first  $P_1 = 0$  then  $P_2 = 0$ . Equations (6) for a laser input can then be solved for  $P_1$  and  $P_2$ , which are then used in eq. (5), to obtain the effective absorptance  $\alpha$ .

Systematic errors arise in this procedure because: (1) The  $I_{ij}$  determined electrically are not exactly those required in the laser measurement. This is another aspect of the non-equivalence of sources problem. (2) The quantities  $F_2$  and  $\alpha_2$  have systematic errors. There will be another systematic error because the mean of our observations will differ from the population mean.

Equations (6) require the choice of a particular time  $t$ . To choose this time, we consider the random errors in the  $I_{ij}$  and a pseudo-reflectance experiment. In table II are shown the values of the electrically determined  $I_{ij}$  and the standard deviations for calorimeter K1-2 at 20 sec intervals after the power ( $P_1$  or  $P_2$ ) is turned on. The values are averages of four determinations. Evidently, the precision of these data improves with time up to 100 sec.

In the pseudo-reflectance experiment, we use simultaneous inputs of electrical power to both the auxiliary calorimeter and the calorimeter proper to check how well eq. (5) reproduces the known ratio  $P_2/(P_1 + P_2)$ . The averages of two experiments are shown in table III. The known value of the pseudo-reflectance  $P_2/(P_1 + P_2)$  is 1.45%. Evidently, the best values are obtained for times of 100 sec or less.

On the basis of the data in tables II and III we select 100 sec as the best time to use for calculating the absorptance. The dual electrical experiment was not carried out for K1-2, but it is very similar to K1-1 both in construction and in the value obtained for the absorptance.

At  $t = 100$  sec, the reflectance at  $10.6 \mu\text{m}$  for calorimeter K1-1 is 1.47% with a standard deviation  $s = 0.045$  based on four measurements. The corresponding values for K1-2 are 1.49% and  $s = 0.030$  based on two measurements. Since the measurement is made with the same auxiliary calorimeter and K1-1 and K1-2 are very similar, we pool the standard deviation to obtain  $s_p = 0.042$  with four degrees of freedom. The 99% confidence intervals are  $1.47 \pm .19\%$  and  $1.49 \pm .19\%$  respectively.

There is another systematic error because the  $I_{ij}$  refer to electrical and not laser inputs. In figure 4 is a plot of  $I_{11}$  from table II. Excepting the first point, these observations lie on a straight line with an intercept at 10 sec, which represents the delay between the heater input and the response of the thermometer. Theoretically, the data represent part of a curve which eventually reaches a constant value for  $I_{11}$ , but here we are only concerned with an adequate representation of the first part of that curve. In figure 2 the heater is located closer to the thermometer than the absorbing surface for laser inputs. A laser ray parallel to the axis and striking the point of the conical mirror will be reflected to the wall 8 cm from the mirror. Rays focused through an aperture in front of the

Table II. Summary of the  $I_{ij}$  values for two sets of four experiments, calorimeter K1-2.

Time	$I_{11}$	s	$I_{21}$	s	$I_{12}$	s	$I_{22}$	s
sec	mV/W	%	mV/W	%	mV/W	%	mV/W	%
20	4.6	11.0	.03	72.6	.08	115.5	1.34	.29
40	13.1	0.9	.18	29.9	.29	28.5	2.61	.19
60	21.3	3.7	.58	13.0	.58	28.4	3.77	.12
80	29.6	3.3	1.30	8.3	1.19	17.2	4.82	.10
100	38.0	1.9	2.41	5.5	2.26	12.3	5.76	.08
120	46.2	1.6	3.92	3.6	3.58	12.0	6.60	.09
140	54.8	1.1	5.86	2.7	5.22	8.2	7.35	.07
160	63.0	1.0	7.94	8.8	7.12	7.3	8.02	.08
180	71.3	1.0	11.04	1.8	9.38	6.2	8.61	.07
200	79.4	0.8	14.24	1.6	11.77	5.7	9.13	.08
220	87.8	0.8	17.83	1.4	14.48	4.6	9.59	.05

Table III. Summary of dual electrical input to K1-1.

Time (sec)	Pseudo-Reflectance
20	1.46%
60	1.48
100	1.46
140	1.54
180	1.59
220	1.63

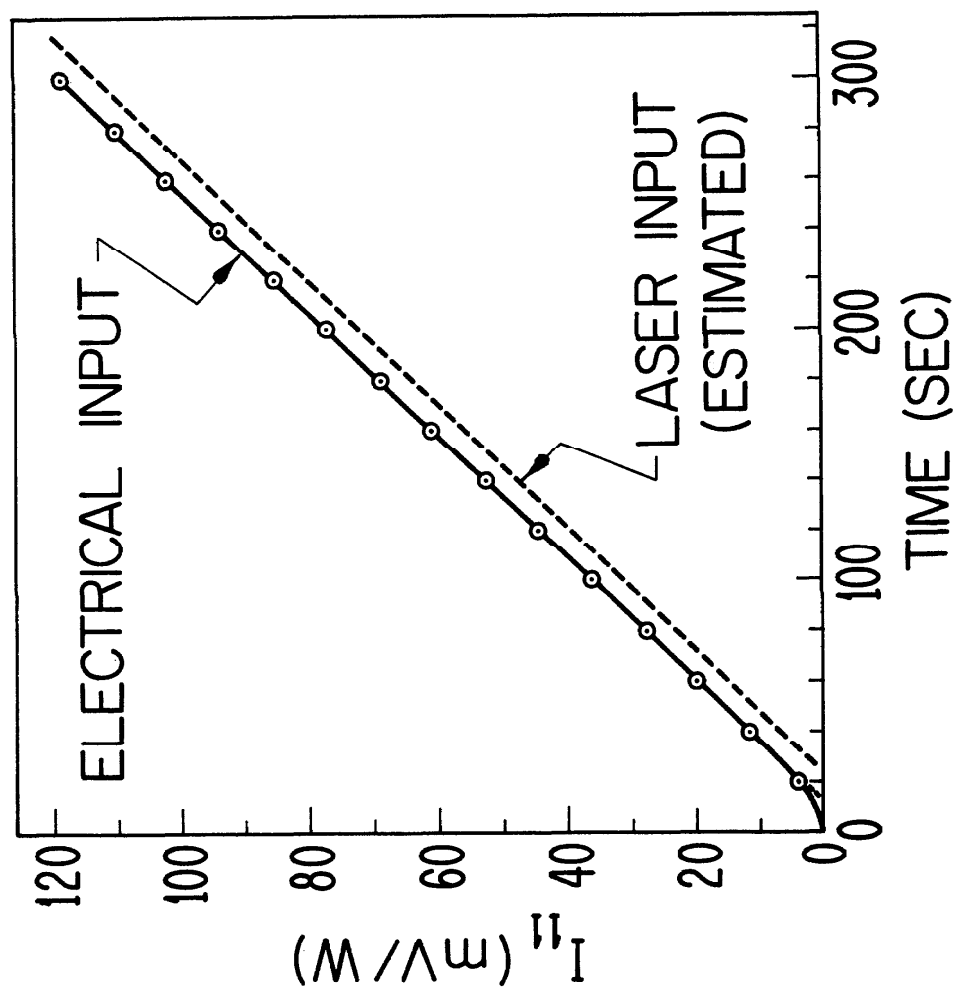


Figure 4. Plot of the function  $I_{11}$  required to determine the effective absorptance. The solid line represents an electrical input. The dotted line represents a possible laser input. The difference between the two causes a systematic error in the absorptance.

calorimeter will strike the mirror at a greater angle of incidence and be reflected to the wall closer to the mirror. A laser input will likely be more remote from the thermometer than an electrical input with a correspondingly greater delay, so that we would expect a line parallel to that for the electrical input but displaced to longer times. Such a line, marked Laser Input (estimated), has been sketched in the figure. Of course, the actual time displacement will depend on the size and divergence of the beam as discussed above. For example, one of our laser measurement experiments with a large beam ( $\sim 1$  cm diameter) gives an intercept of 40 sec for a plot of  $T_1 - T_1(\infty)/P_1$ . Taking a line with the same slope as  $I_{11}$  but displaced by 10 sec, we find a constant difference of 5 mV/W between  $I_{11}$  determined electrically and what it might be for a small divergent beam. This smaller value of  $I_{11}$  gives 1.24% for the effective reflectance at  $t = 100$  sec, compared to 1.46% for the electrically determined  $I_{11}$ .

To explore this problem further, we compared data taken with a different electrical heater on calorimeter 2, figure 1 [8]. The heater used to obtain most of the data was wound on the cylindrical surface. A second heater was fastened to the flat surface, which would be closer to the laser input. The values of  $I_{22}$  obtained with this heater were then used with the original  $I_{11}$  to obtain the effective absorptance ( $\alpha = 1.38\%$ ) and with the decreased values of  $I_{11}$  to obtain  $\alpha = 1.19\%$ . Considering all cases, the calculated effective absorptance ranges from 1.19 to 1.46%. Although both of these values are less than 1.46%, we consider that the error might also be of the other sign and allow a systematic error of 0.2% for the bias in the  $I_{ij}$ .

We must also allow for the fact that the laser beams used were small and covered only the central part of the mirror. A larger beam would be reflected to the walls nearer the mirror and might give a different value for the absorptance. We estimate this error to be less than 0.2%. A better procedure for obtaining a random sample would give a better evaluation of this error.

To summarize the systematic error in the absorptance measurement for each calorimeter, we estimate  $\pm 0.19\%$  from the random error of the absorptance measurement and  $\pm 0.4\%$  for the sum of the error in the  $I_{ij}$  and in the sampling.

## 8. COMPARISON OF K1-1 AND K1-2

When two calorimeters are used to measure a beam ratio as in figure 1, the ratio is biased because of the systematic errors  $(\delta_1, \delta_2)$  in energy measurement by each calorimeter. The true beam ratio can be obtained by interchanging the calorimeter sub-systems so the bias between calorimeter sub-systems can then be calculated. In this instance each calorimeter sub-system includes a K-1 calorimeter, a temperature controller, a constant current supply, a signal amplifier, and a digital voltmeter that reads the response of the temperature bridge. In one position the beam ratio  $R_1$  is the ratio of energies to the two calorimeters

$$R_1 = \frac{E_1(1 + \delta_1)\Delta T_1}{E_2(1 + \delta_2)\Delta T_2} \quad (7)$$

where the E's are the calibration factors and the  $\Delta T$ 's are the corrected temperature rises. Reversing the calorimeters, the second ratio  $R_2$  is

$$R_2 = \frac{E_2(1 + \delta_2)\Delta T_2'}{E_1(1 + \delta_1)\Delta T_1'} \quad (8)$$

where the primes are used to indicate that the temperature rises will be different in the two experiments. The true ratio  $R$  is equal to  $R_1$  or  $R_2$  corrected for the errors, that is

$$R = \frac{E_1\Delta T_1}{E_2\Delta T_2} = \frac{E_2\Delta T_2'}{E_1\Delta T_1'} \quad (9)$$

Multiplying expressions from eq. (7) and (8) we obtain

$$R_1 R_2 = \frac{E_1(1 + \delta_1)\Delta T_1}{E_2(1 + \delta_2)\Delta T_2} \frac{E_2(1 + \delta_2)\Delta T_2'}{E_1(1 + \delta_1)\Delta T_1'} \quad (10)$$

which simplifies to

$$R_1 R_2 = \frac{\Delta T_1}{\Delta T_2} \frac{\Delta T_2'}{\Delta T_1'} \quad (11)$$

that is, the product  $R_1 R_2$  is independent of the calibration factor and its associated error. Multiplying together the two expressions for  $R$  in eq. (9) we obtain

$$R^2 = \frac{\Delta T_1}{\Delta T_2} \frac{\Delta T_2'}{\Delta T_1'} \quad (12)$$

By inspection of eqs. (11) and (12)

$$R = \sqrt{R_1 R_2} \quad . \quad (13)$$

Substitution in eqs. (7) from (9) gives

$$R_1 = R(1+\delta_1)/(1+\delta_2) \quad , \quad (14)$$

and similarly from eqs. (8) and (9)

$$R_2 = R(1+\delta_2)/(1+\delta_1) \quad . \quad (15)$$

From eqs. (14) and (15)

$$R_1 - R_2 = R \left\{ \frac{1 + \delta_1}{1 + \delta_2} - \frac{1 + \delta_2}{1 + \delta_1} \right\} \quad , \quad (16)$$

which simplifies to

$$\frac{R_1 - R_2}{R} = \frac{2\delta_1 + \delta_1^2 - 2\delta_2 - \delta_2^2}{(1 + \delta_1)(1 + \delta_2)} \quad . \quad (17)$$

For K1-1 and K1-2,  $\delta < 0.02$  to  $0.03$ , so that we can use the approximations  $\delta^2 \ll \delta$  and  $\delta \ll 1$  to obtain

$$\frac{R_1 - R_2}{R} = 2(\delta_1 - \delta_2) \quad . \quad (18)$$

This equation is the basis for deducing the difference in the systematic errors of the two calorimeter sub-systems from intercomparisons using a beam splitter.

The calorimeter sub-systems of K1-1 and K1-2 have been intercompared using a gallium arsenide beam splitter. Twenty-five measurements with K1-2 as monitor give an average ratio of 1.7919 and a standard deviation of 0.0143. Seventeen measurements with K1-1 as monitor give an average of 1.8154 and a standard deviation of 0.0142. From eq. (18) we calculate a difference in systematic errors of 0.65%. This difference is large enough that we accept the statistical hypothesis that there is a real systematic difference between the two calorimeters at the 99% confidence level. The accuracy of the digital voltmeters are regularly monitored at NBS by equipment that is traceable to the international system of units. The scale errors of these digital voltmeters are below .01%; therefore their contributions to the systematic difference of the sub-systems are insignificant in comparison to the total systematic difference.



## 9. ERROR OF A LASER ENERGY MEASUREMENT

The over-all error is obtained by the methods of combining errors discussed in [6].

The precision of a laser energy measurement is taken to be the same as that of an electrical calibration as calculated by eq. (4). This procedure is justified by comparing the precision of electrical and beam splitter experiments. Franzen [1] has made beam ratio measurements using K1-1 and K1-2 with a gallium arsenide beam splitter. For a set of twenty-five measurements with K1-1 as calorimeter 1, figure 4, and K1-2 as calorimeter 2, Franzen observed the standard deviation of the beam ratio to be 0.80%. The average energy to K1-1 was 867 J and to K1-2 481 J. Comparing the standard deviations in per cent from these energies according to eq. (4), we get 0.42% and 0.74% respectively for K1-1 and K1-2. Combining these we get 0.85% as the estimated standard deviation of a beam splitter measurement. The observed standard deviation (0.80%) and that calculated from the electrical calibrations (0.85%) do not differ significantly (F test [6]) and we reject the hypothesis that the precision of a laser energy measurement is different from the precision of an electrical measurement.

The systematic errors are combined in two different ways, depending on how they were estimated. Those determined by a sampling process are combined by root-mean-square; those estimated from calculations or otherwise are summed. Although summing is a pessimistic method of combining errors, the evidence is that root-mean-square combination of systematic errors of this kind leads to over-optimistic error statements [6].

For calorimeter K1-1, the error due to the non-equivalence of sources is estimated from the difference between the calibration factors with the regular and the mirror heater to be  $.0001 \pm .0241$  at the 99% confidence level. Taking the sum as a conservative estimate, we get  $.0242 : 2.2609 = 1.07\%$ . The random component of error in the absorptance  $\alpha$  is 0.19%. These two are combined by root-mean-square to give 1.08%. The estimated limits of systematic error in the electrical calibrations is  $\pm 0.4\%$  and the estimates for the absorptance error are 0.2% in the  $I_{ij}$  and 0.2% for insufficient variation of the incident beam location. The combined estimated limits of systematic error are  $\pm (1.08 + 0.4 + 0.2 + 0.2) = \pm 1.9\%$  for calorimeter K1-1.

For calorimeter K1-2, the non-equivalence error is  $0.0057 \pm .0204$  or 1.21%. Combining with the 0.19% from the absorptance measurement, we get 1.22%. The other estimates are the same as for K1-1, leading to a combined estimate of  $\pm 2.0\%$ .

We obtain the limits of error of a single measurement of energy by adding to the estimated limits of systematic error the product  $ts$ , where the  $t$  statistic is chosen for the 99% confidence level and the number of electrical calibration experiments less one and the standard deviation  $s$  is calculated by eq. (4) from the standard deviation  $s_{\Delta T}$  also obtained from the electrical calibration data. For a 1 kJ measurement with calorimeter K1-1,  $Es_{\Delta T}/W$  is  $2.262 \times 3.65 \div 1000 \times 100 = 0.82\%$  and  $t = 2.7$  for 35 degrees of freedom and 99% confidence interval. The estimated limits of error are  $\pm (2.7 \times 0.82 + 1.9) = \pm 4.1\%$ . The corresponding numbers for K1-2 are 0.81% and 4.2%.

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